# Assessing profit margins

When setting the price of a new product in the market, how can a business strike a balance between attracting consumers to buy its products and earning as much profit per unit as possible? In this lesson, students will investigate how mathematics can be used in the business world to set appropriate prices.

Prerequisite knowledge: students must be familiar with the components of a quadratic equation such as vertex, zeroes, etc. Students must also know how to "complete the square". They should also have prior familiarity with the technology being used by the teacher.

mind	Subject Suggested timing	MPM2D - Principles of Mathematics 70 minutes (If reviewing the use of technology, or to allow for more discussion in the consolidation component, this would be closer to 2 periods)
	Financial literacy objectives	<ul> <li>At the end of this lesson, students will:</li> <li>understand how a business sets the price of a product;</li> <li>describe factors that could impact the price of a product.</li> </ul>
Curriculum expectations	<ul> <li>Mathematics, grades 9 and 10 (2005 revised)</li> <li>Principles of Mathematics (MPM2D)</li> <li>Quadratic relations of the form y=ax<sup>2</sup> +bc+c</li> <li>Solve problems arising from a realistic situation represented by a graph or an equation of a quadratic relation, with and without the use of technology.</li> </ul>	
Assessment	Collect student worksheets.	
What you need	<ul> <li>Shnark Industries, Ltd worksheet (Appendix A)</li> <li>Appropriate technology (Appendix B)</li> <li>TI graphing calculator</li> </ul>	
Minds on	Teacher opens with the following consideration: "When a business introduces a new product into the market, why is the price they set important?" The discussion should lead students to understand that price affects whether a consumer will purchase the product and it affects the net profit of the business.	

PAGE 1

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GRADE



Minds on (cont'd)

Explain to students that having an understanding of the mathematics of how pricing works can support a retailer in ensuring he/she is not overvaluing or undervaluing products sold to consumers.

#### Think-pair-share

- Prompt partners to consider 2 pricing techniques used by retailers: pricing products above the market (high) and below the market (low). What would they gain and lose in each situation? Instruct students to share their comments with a partner.
- Help students to understand that a product that is priced too low can gain more consumers (market share) but will earn less profit per unit. On the contrary, a product that is priced too high can gain a higher profit per unit but may lose consumers (market share).
- Discuss how mathematics can be used to determine a price that balances what a business needs to ensure it covers the product's costs and makes a profit with fair and equitable price points for the consumer.

#### Context for learning

Bill Fold is a character who is constantly getting himself into financial scrapes. Use the scenario below to provide students with a context for learning.

Bill Fold has an idea for a new product – designer decals for rubber boots. Bill can find someone to make the decals but he's not sure how much he would charge for the decals. What's the optimal price he could charge to maximize profit?

#### Action

## Instructions (If necessary, review the instructions for using the selected technology-see Appendix B)

- Distribute copies of the worksheet (Appendix A).
- Draw/show a downward-opening parabola with price represented on the x-axis, and profit represented on the y-axis.
  - Ask for a volunteer to write the general equation of a parabola in vertex form  $y = a(x-h)^2 + k$  on the board.
  - Ask another student to identify the point on the parabola which maximizes profit and note the variable coordinates of this point according to the general equation (h, k).
  - Invite students to discuss in a Think-pair-share activity what each variable in the formula (*a, k, h, x, y*) represents in terms of a business curve.
- Ask for a volunteer to read the introduction of Shnark Industries, Ltd. aloud to the class (Appendix A).
- Students should complete the given worksheet, using the selected technology.



### Assessing profit margins



Consolidation/
debrief

Discuss with students some of the following questions:

- 1. Identify the challenges a business owner might find in the process of calculating the selling price of a product using quadratic relations.
- 2. What are the variables that could affect the vertex of price versus profit graph of a product over time?
- 3. Assess the process for determining profit based on selling a product. What questions could be raised by a graph that did not go through the origin?
- 4. A business knows how much it must charge in order to make a profit. What factors might a consumer consider to determine if the price of the product is "reasonable"?
- 5. Is there any additional or "missing" information that would help in completing your analysis, in particular to address the next two questions?
- 6. At your ideal price and profit, how will you know how many units are sold?
- 7. If you had set the price at \$25 per unit, how much profit would you have made and how many units would you have sold?

Have the students summarize the discussion in their journals or as homework.



### Assessing profit margins



### Shnark Industries, Ltd. - worksheet

You are the CEO of Shnark Industries, Ltd. and have developed a device called *The Hax*. When plugged into any portable media player, *The Hax* removes unnecessary files and malware and redistributes data to clear up room for more files!

Before large retailers will take you on, you must set up booths at trade shows and sell *The Hax* directly to customers. The success of your company will be based on not only how many units you sell but on your product price. You will use the given information and your knowledge of quadratics to determine:

- At what price should you sell The Hax to gain maximum profit?
- If the product price would be acceptable (and perceived to be fair) to consumers?
- 1. Sketch a graph of price per unit (x) vs. profit (y) and label your axes: Keep in mind:
- Setting the price too high means higher profit per unit sold but not as many units sold.
- Setting the price too low means lots of units sold but may lower overall profit.

2. Write down the general equation for a quadratic function in vertex form  $y = a(x-h)^2 + k$ .





### Shnark Industries, Ltd. – worksheet (cont'd)

3. A market research firm you hired analyzes data and finds the equation of your curve is  $y = -2x^2 + 60x$ .

Convert the equation into vertex form (check your notes from the last unit if you cannot remember how to do this).

- 4. What do the following represent for your business curve?
  - a: h: k: y: x:
- 5. Now you can determine the ideal price to set for your unit as well as the profit you will make. What are these values and how did the equation help you determine them?
- 6. Using appropriate technology, graph the equation given in question 3.





### Shnark Industries, Ltd. – worksheet (cont'd)

- 7. Does the graph's vertex agree with what you determined from your equation?
- 8. Can you answer the question "At what price will no one purchase your product?"
- 9. Are there any reasons why a business might choose to set a price other than the ideal price to maximize profit?



### APPENDIX B

### Assessing profit margins



### Appropriate technology

Hints for using Technology: Graphing calculators (TI83/84, Nspire), Gizmos, Geometers Sketchpad, the CLIPS graphing calculator tool, and Wolfram Alpha.

#### Part 1: Using TI 83(84)/TI-Nspire Graphing calculators

**Step 1:** Click on y= and enter the equation  $y = -2x^2 + 60x$ .

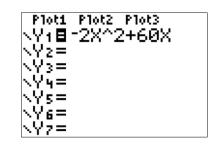
Your screen should resemble

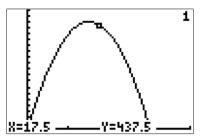
(Note: make sure any other graphing options (e.g. **STATPLOT**) are cleared)

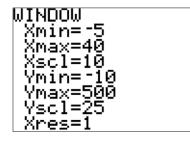
**Step 2:** Use the **TRACE** button to answer the questions

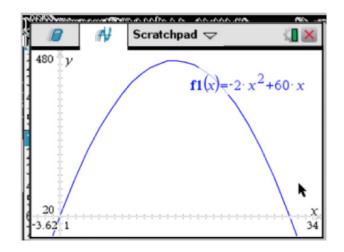
To display the graph optimally, click on the **WINDOW** button and enter the setting shown.

**Step 3:** Entering the equation and displaying the graph on the TI-Nspire is an easier process and the display will be similarbut contains more numeric data.









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## Appropriate technology (cont'd)

## Part 2: Using Gizmos (Quadratic in Polynomial Form-Activity A)

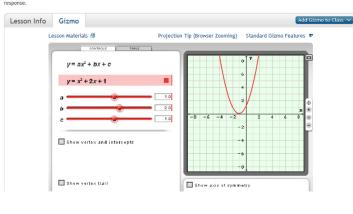
#### www.explorelearning.com/ontario

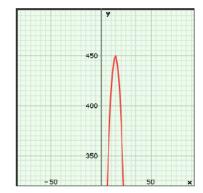
This Gizmo is very well suited conceptually to this activity, allowing the student to easily change the values of the coefficients of the parabola and observe the effect on the characteristics of the curve, and their interpretation in the application.

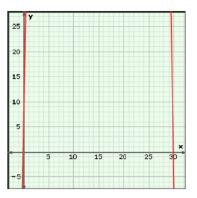
NOTE: the sliders do not allow for coefficients as large as are needed, however the numbers can be typed directly in to the fields next to the sliders, and the graph zoomed out accordingly. Extra detail can be recorded by zooming back in on portions of the graph.The axis of symmetry can also be added, which would further identify the vertex

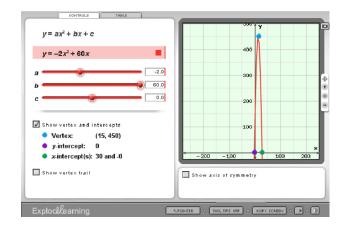
#### Quadratics in Polynomial Form - Activity A

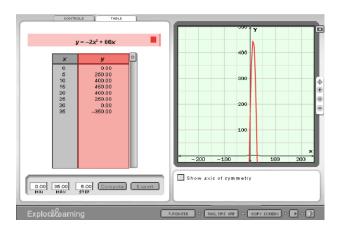
Compare the graph of a quadratic to its equation in polynomial form. Vary the coefficients of the equation and explore how the graph changes in







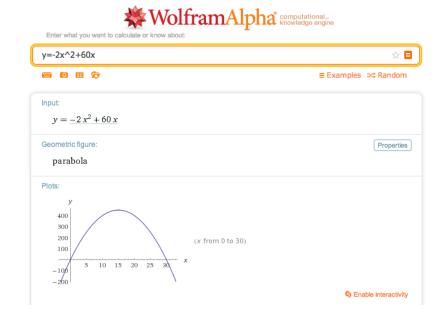




# Appropriate technology (cont'd)

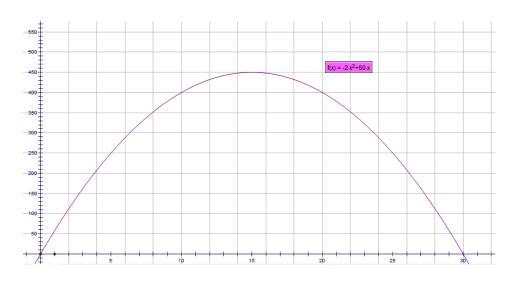
#### Part 3 : Using Wolfram Alpha

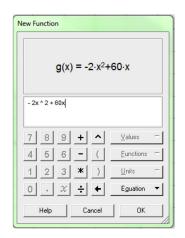
For schools familiar with the Wolfram Alpha website, that too could easily be used in this situation.



#### Part 4 : Using Geometer's Sketchpad

NOTE using rectangular coordinates instead of a square grid allows for easier independent scaling of the two axes.





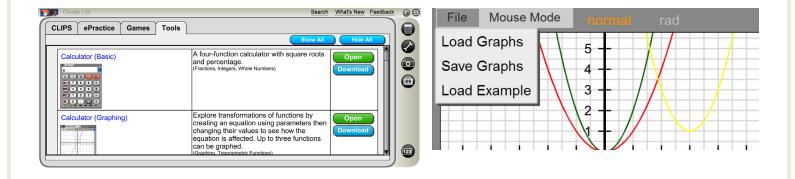


## Appropriate technology (cont'd)

#### Part 5: Using the CLIPS (MathGAINS) graphing calculator tool

#### www.mathclips.ca

Similar to the Gizmo, this allows for easy changes to the coefficients and the corresponding immediate observation of the effect on factors that relate to the model. It may be helpful to load the quadratic example and then modify the parameters and window settings as shown.



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Note: You can send students and parents dire to CLIPS via www.mathclips.ca

Math CLIPS Evidence - November 2011 Use of Critical Learning Instructional Paths Supports (CLIPS) can increase student learning and influence bacher creation.

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