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| About this Lesson |
| When setting the market price of a new product, how can a business strike a balance between  attracting consumers to buy its products and earning as much profit per unit as possible? |

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| **Grade Level** | **Course(s)/subject(s)** | **Learning Goal(s)** | **Suggested**  **Timing** |
| 10  11–12 | MPM2D – Principles of Mathematics  MCR3U – Functions  MCF3M – Functions and Applications  MBF3C – Foundations for College Mathematics  MAP4C – Foundations for College Mathematics  BDI3C – Entrepreneurship:  The Venture  BMI3C – Marketing: Goods, Services, Events | At the end of this lesson, students will:   * understand how a business sets the price of a product * describe factors that could impact the price of a product | 70-140 minutes  *(depending on possible need to review use of technology)* |

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| Curriculum Links |
| [Mathematics, grades 9 and 10 (2005 revised)](http://www.edu.gov.on.ca/eng/curriculum/secondary/math910curr.pdf)  Principles of Mathematics (MPM2D)  Solving Problems Involving Quadratic Relations  Solve problems arising from a realistic situation represented by a graph or an equation of a quadratic relation, with and without the use of technology.  Foundations of Mathematics (MPM2P)  Solving Problems by Interpreting Graphs of Quadratic Relations  Solve problems involving a quadratic relation by interpreting a given graph or a graph generated with technology from its equation. |

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| Curriculum Links (cont’d.) |
| [Mathematics, Grade 11 - 12](http://www.edu.gov.on.ca/eng/curriculum/secondary/math1112currb.pdf)  Functions (MCR3U)  2.3 By the end of the course, students will:   * solve problems involving quadratic functions arising from real-world applications and represented using function notation   *Sample problem:* The profit, P(x), of a video company, in thousands of dollars, is given by: P(x) = –5x2 + 550x – 5000, where x is the amount spent on advertising, in thousands of dollars. Determine the maximum profit that the company can make, and the amounts spent on advertising that will result in a profit and that will result in a profit of at least $4,000,000.  Functions and Applications (MCF3M)  1.5 By the end of the course, students will:   * determine, through investigation, and describe the connection between the factors used in solving a quadratic equation and the x-intercepts of the graph of the corresponding quadratic relation   *Sample problem:* The profit, P, of a video company, in thousands of dollars, is given  by: P(x) = –5x2 + 550x – 5000, where x is the amount spent on advertising, in thousands of dollars. Determine, by factoring and by graphing, the amount spent on advertising that will result in a profit of $0. Describe the connection between the two strategies.  Foundations for College Mathematics (MBF3C)  1.2 By the end of the course, students will:   * determine and interpret meaningful values of the variables, given a graph of a quadratic relation arising from a real-world application   *Sample problem:* Under certain conditions, there is a quadratic relation between the  profit of a manufacturing company and the number of items it produces. Explain how  you could interpret a graph of the relation to determine the numbers of items produced for which the company makes a profit and to determine the maximum profit the company can make.  Foundations for College Mathematics (MAP4C)  2.1 By the end of this course, students will:   * interpret graphs to describe a relationship (e.g., distance travelled depends on driving time, pollution increases with traffic volume, maximum profit occurs at a certain sales volume), using language and units appropriate to the context |

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| Curriculum Links (cont’d.) |
| [Business, Grade 11–12](http://www.edugains.ca/newsite/curriculum/secondaryresources/business2.html)  Entrepreneurship: The Venture (BDI3C)  The Financial Plan  By the end of this course, students will:   * analyse financial goals that an entrepreneur might establish for a new business venture (e.g., break-even point, projected profit levels, return on investment, market share)   Marketing: Goods, Services, Events (BMI3C)  Price   * determine the price for a product, using relevant financial data (e.g., calculate total cost, break-even point, selling price, gross profit margin) |

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| Inquiry Question |
| When a business introduces a new product into the market, why is the price they set important? |

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| Materials List |
| * Shnark Industries, Ltd. – Worksheet (Appendix A) * Access to Appropriate Technology (Appendix B) |

| **Timing**  (Mins.) | **Lesson Sequence** | **Assessment for and as Learning Opportunities** (self/peer/teacher) |
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| MINDS ON | | |
|  | Teacher opens with the following consideration: “When a business introduces a new product into the market, why is the price they set important?” The discussion should lead students to understand that price affects whether a consumer will purchase the product and it affects the net profit of the business.  Explain to students that having an understanding of the mathematics of how pricing works can support a retailer in ensuring they are not overvaluing or undervaluing products sold to consumers. |  |

| **Timing**  (Mins.) | **Lesson Sequence** | **Assessment for and as Learning Opportunities** (self/peer/teacher) |
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| MINDS ON (cont’d.) | | |
|  | Think-pair-share   * Prompt partners to consider two pricing techniques used by retailers: pricing products above the market (high) and below the market (low). What would they gain and lose in each situation? Instruct students to share their comments with a partner. * Help students to understand that a product that is priced too low can gain more consumers (market share) but will earn less profit per unit. On the contrary, a product that is priced too high can gain a higher profit per unit but may lose consumers (market share). * Discuss how mathematics can be used to determine a price that balances what a business needs to ensure it covers the product’s costs and makes a profit with fair and equitable price points for the consumer. |  |
|  | Context for Learning  Bill Fold is a character who is constantly getting himself into financial scrapes. Use the scenario below to provide students with a context for learning.  Bill Fold has an idea for a new product – designer decals for rubber boots. Bill can find someone to make the decals but he’s not sure how much he would charge for the decals. What’s the optimal price he could charge to maximize profit? |  |

| **Timing**  (Mins.) | **Lesson Sequence** | **Assessment for and as Learning Opportunities** (self/peer/teacher) |
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| ACTION | | |
|  | Instructions  *(If necessary, review the instructions for using the selected technology – see Appendix B)*   * Distribute copies of the worksheet (Appendix A). * Draw/show a downward-opening parabola with price represented on the x-axis, and profit represented on the y-axis.   + Ask for a volunteer to write the general equation of a parabola in vertex form **y = a(x-h)2 + k** on the board.   + Ask another student to identify the point on the parabola which maximizes profit and note the variable coordinates of this point according to the general equation *(h, k)*.   + Invite students to discuss in a Think-pair-share activity what each variable in the formula *(a, k, h, x, y)* represents in terms of a business curve. * Ask for a volunteer to read the introduction of Shnark Industries, Ltd. aloud to the class (Appendix A).   Students should complete the given worksheet, using the selected technology. |  |

| **Timing**  (Mins.) | **Lesson Sequence** | **Assessment for and as Learning Opportunities** (self/peer/teacher) |
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| CONSOLIDATION/DEBRIEF | | |
|  | Discuss with students some of the following questions:   1. Identify the challenges a business owner might find in the process of calculating the selling price of a product using quadratic relations. 2. What are the variables that could affect the vertex of price versus profit graph of a product over time? 3. Assess the process for determining profit based on selling a product. What questions could be raised by a graph that did not go through the origin? 4. A business knows how much it must charge in order to make a profit. What factors might a consumer consider to determine if the price of the product is “reasonable”? 5. Is there any additional or “missing” information that would help in completing your analysis, in particular to address the next two questions? 6. At your ideal price and profit, how will you know how many units are sold? 7. If you had set the price at $25 per unit, how much profit would you have made and how many units would you have sold? | Have the students summarize the discussion in their journals or as homework |

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| Shnark Industries, Ltd. – Worksheet |
| You are the CEO of Shnark Industries, Ltd. and have developed a device called *The Hax*. When plugged into any portable media player, *The Hax* removes unnecessary files and malware and redistributes data to clear up room for more files!  Before large retailers will take you on, you must set up booths at trade shows and sell *The Hax* directly to customers. The success of your company will be based on not only how many units you sell but on your product price. You will use the given information and your knowledge of quadratics to determine:   * At what price should you sell *The Hax* to gain maximum profit? * If the product price would be acceptable (and perceived to be fair) to consumers?  1. Sketch a graph of price per unit (x) vs. profit (y) and label your axes. Keep in mind:  * Setting the price too high means higher profit per unit sold but not as many units sold. * Setting the price too low means lots of units sold but may lower overall profit.  1. Write down the general equation for a quadratic function in vertex form **y = a(x-h)2 + k.** 2. A market research firm you hired analyzes data and finds the equation of your curve is **y = – 2x2+ 60x**.   Convert the equation into vertex form (check your notes from the last unit if you cannot remember how to do this).   1. What do the following represent for your business curve?   a:  h:  k:  y:  x:   1. Now you can determine the ideal price to set for your unit as well as the profit you will make. What are these values and how did the equation help you determine them? 2. Using appropriate technology, graph the equation given in question 3. 3. Does the graph’s vertex agree with what you determined from your equation? 4. Can you answer the question “At what price will no one purchase your product?”? 5. Are there any reasons why a business might choose to set a price other than the ideal price to maximize profit? |

**APPENDIX A**

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| Appropriate Technology |
| Hints for using Technology: Graphing calculators (TI83/84, Nspire), Gizmos, Geometers Sketchpad, the CLIPS graphing calculator tool (not shown explicitly), Desmos.  Part 1: Using TI 83(84)/TI-Nspire Graphing calculators  **Step 1:** Click on *y*= and enter the equation *y = – 2x2+ 60x.*  Your screen should resemble:  (Note: make sure any other graphing options (e.g. STATPLOT)  are cleared)    **Step 2:** Use the TRACE button to answer the questions  To display the graph optimally, click on the WINDOW button and enter the setting shown.  **Step 3:** Entering the equation and displaying the graph on the TI-Nspire is an easier process and the display will be similar but contains more numeric data. |

**APPENDIX B**

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| Appropriate Technology (cont’d.) |
| Part 2: Using [Gizmos](http://www.explorelearning.com/ontario) (Quadratic in Polynomial Form)    This Gizmo is very well suited conceptually to this activity, allowing the  student to easily change the values of the coefficients of the parabola and observe the effect on the characteristics of the curve, and their interpretation in the application.  NOTE: the sliders do not allow for coefficients as large as are needed, however the numbers can be typed  directly in to the fields next to the sliders, and the graph zoomed out  accordingly. Extra detail can be  recorded by zooming back in on  portions of the graph. The axis of  symmetry can also be added,  which would further identify the vertex. |

**APPENDIX B**

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| Appropriate Technology (cont’d.) |
| Part 3: Using Geometer’s Sketchpad  **NOTE:** Using rectangular coordinates instead of a square grid allows for easier independent scaling of the two axes.    Part 4: Using [Desmos](https://www.desmos.com/calculator) |

**APPENDIX B**

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| Appropriate Technology (cont’d.) |
| Part 5: Using the CLIPS (MathGAINS) graphing  calculator tool  Main page: [www.mathclips.ca](http://www.mathclips.ca)  Learning Tools : <http://mathies.ca/learningTools.php>  CLIPS support:  <http://www.edugains.ca/newsite/math/clips.html>  Similar to the Gizmo, this allows for easy changes  to the coefficients and the corresponding immediate  observation of the effect on factors that relate to the  model. It may be helpful to load the quadratic example and then modify the parameters and window settings as shown. |

**APPENDIX B**